Incorporating Point Basal Area, Tree Dominance, Soil Type and Altitude Into the Existing Taper Models for *Eucalyptus pilularis*

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Abstract The objectives of this study were to assess and possibly, improve the performance of the existing taper models for *Eucalyptus pilularis* (Blackbutt), by incorporating soil type, point basal area, altitude and tree dominance into model predictions. The analyses were based on six taper models described in a study by Muhairwe (1995), and involved applying the models to the data collected for this study and estimating new model parameters based on this data. The results indicated that model predictions were both biased and imprecise, due to the fact that the method used to estimate model parameters did not capture all of the relevant data characteristics. In particular, this method could not take into account the fact that some stem diameters were measured along the same tree, and were therefore related. This attribute of the data was captured through the process of two-stage modelling, which involves fitting a model to each tree separately and relating the resulting parameters to tree and stand characteristics. The results of this study indicate that this process provides more appropriate estimates of model parameters, improving both bias and precision of model predictions. In addition to being a more appropriate method of describing correlated data, two-stage modelling allows for an accurate assessment of the effects of soil type, altitude, point basal area and tree dominance on both form and taper of *E.pilularis*. The results of this study indicate that the use of two-stage modelling to incorporate each of these factors into the existing taper models for *E.pilularis* is likely to improve their stem diameter predictions. This process can also increase model performance from a practical viewpoint, by allowing a single taper model to be applied to a wider range of circumstances. To achieve this practicality, a wider range of data is necessary, so further research in this area is recommended.

1. INTRODUCTION

To date, taper research on *Eucalyptus* species has been less extensive than that of other hardwood species and especially softwoods. Taper research on *E.pilularis* is perhaps the most recent - several models were developed by Carter (1994) and modified in 1995 by Muhairwe. In addition, Muhairwe (1995) developed three nonlinear models consisting of a single taper equation with a variable exponent - termed variable-form taper models. The variable form approach differs from other methods of taper modelling, because it utilises a single continuous function which "*...describes the shape of the bole, with a changing exponent from ground to top to compensate for the neiloid, paraboloid, and conic forms.*" (Kozak, 1988, p.1363). In 1993, a study by Kozak and Smith revealed that variable-form taper equations were highly suitable for describing the form of a tree stem, because they use "*...a changing exponent from ground to top... to describe the gradually changing stem profile...*", while the exponent itself "*...contains the ratio of DBH and height, which explains a significant amount of tree to tree variation in form.*" (Kozak and Smith, 1993, p.443).

In his study, Muhairwe pointed out the need for further improving the existing taper models for *E.pilularis*, in terms of both practical and statistical considerations (see Kozak and Smith, 1993). In particular, he suggested that the models' practicality could be improved by incorporating the effects of stand attributes into model predictions, while an improvement in model accuracy could be achieved through the use of a more appropriate sample - for instance by measuring more than three underbark and overbark diameters per stem, across a wide range of DBH classes obtained through stratified sampling (Philip, 1994). In view of these recommendations, the objectives of this study were to: (1) test the prediction accuracy of the existing *E.pilularis* taper models; and (2) try and improve model performance by incorporating into the analysis factors such as tree dominance, point basal area, soil type and altitude.

2. MATERIALS AND METHODS

The data used in this research were obtained from a sample of sixty *Eucalyptus pilularis* (Blackbutt) trees, located on the North Coast of New South Wales, in State Forests Districts of Dorrigo and Urunga. A larger sample size could not be obtained due to the cost and time constraints of the project. The total number of trees sampled was divided equally between *high* (800m) and *low* (20-300m) altitudes, and between *earths* and *podzolic* soil types at each altitude. Tree dominance was estimated visually, according to four classes (dominant, co-dominant, sub-dominant, suppressed) while soil type and altitude were determined from appropriate area maps.

The models tested were:

(1)

$$dub = a_0 * dbh^{a_1} * (TH - H)^{\left(b_0 + b_1 * \frac{H}{TH} + b_2 * \frac{TH}{H} + b_3 * \frac{dbh}{TH} + b_4 * \left(\frac{dbh}{TH}\right)^2 + b_5 * dbh\right)}$$

(2)

$$dub = a_0 * dbh^{a_1} * \left(\frac{TH - H}{TH - BH}\right)^{\left(b_0 + b_1 * \frac{H}{TH} + b_2 * \left(\frac{H}{TH}\right)^2 + b_3 * \left(\sqrt{\frac{H}{TH}}\right) + b_4 * \frac{dbh}{TH} + b_5 * dbh\right)}$$

(3)

$$dub = a_0 * dbh^{a_1} * a_2^{dbh} * \left(1 - \sqrt{\frac{H}{TH}}\right)^{\left(b_1 * \left(\frac{H}{TH}\right)^2 + b_2 * \frac{TH}{H} + b_3 * \frac{dbh}{TH} + b_4 * TH + b_5 * dbh\right)}$$

(4)

$$dub = b_0 + b_1 * dbh + b_2 * \left(\frac{H - 13}{H}\right) + b_3 * (H - 13)^2 + b_4 * dbh * \left(\frac{H - 13}{H}\right) +$$
$$+ b_5 * dbh * (H - 13) * H + b_6 * dbh * (H - 13)^2$$

(5)

$$dub = b_0 + b_1 * dbh + b_2 * \ln(dbh) + b_3 * H + b_4 * H^4 + b_5 * H^5 + b_6 * \frac{dbh}{H} +$$
$$+ b_7 * \left(\frac{dbh}{H}\right)^2 + b_8 * \frac{H}{dbh}$$

(6)

$$dub = a_0 * dbh^{a_1} * \left(\frac{BH}{H}\right)^{\left(b_0 + b_1 * \left(\frac{1}{H}\right)^2 + b_2 * H^5 + b_3 * \frac{dbh}{H} + b_4 * \frac{H}{dbh}\right)}$$

as described in Muhairwe's (1995) study, where:
dub=diameter under bark at height (H) above ground (cm)
dbh=diameter at breast height overbark (cm)
H=height above ground of each measurement (m)
TH=total tree height (m)
BH=breast height (1.3m)
and a_0, a_1, a_2, b_0, b_1, b_2, b_3, b_4 and b_5 are parameters to be estimated.

Initial tests of model performance were carried out using parameter estimates provided in Muhairwe's (1995) study. Each of the models was then fitted to the data obtained for this study, to obtain new parameter estimates, and model performance before and after fitting was compared. The effect of point basal area, soil type, altitude and tree dominance on model predictions was assessed based on the distribution of model errors (residuals), as well as two-stage modelling. In the second stage of this process, model parameters were related to tree and stand characteristics using the standard least squares procedure (Mendenhall and Beaver, 1990). A two-stage modelling study by Newberry and Burkhart (1985) indicated that the least squares procedure results in similar estimates of model parameters as random function analysis, despite the fact that "*...the errors between the slope and form parameter estimation equations* [are] *correlated.*" (Newberry and Burkhart, 1985, p.111) - whereas they are assumed to be uncorrelated in the least squares procedure.

3. RESULTS AND DISCUSSION

The results of applying each of the models (1)-(6) to the new data, indicated that model predictions were both biased and imprecise. Furthermore, the process of fitting the models to the data, to obtain a new set of parameters, failed to significantly improve their performance (see Figure 1). The possible causes of this poor model performance were that: (1) the form of the models was inappropriate; (2) the data on which the models were based and that used in this study were different; and (3) the method used to estimate model parameters did not capture all of the relevant characteristics of the data.

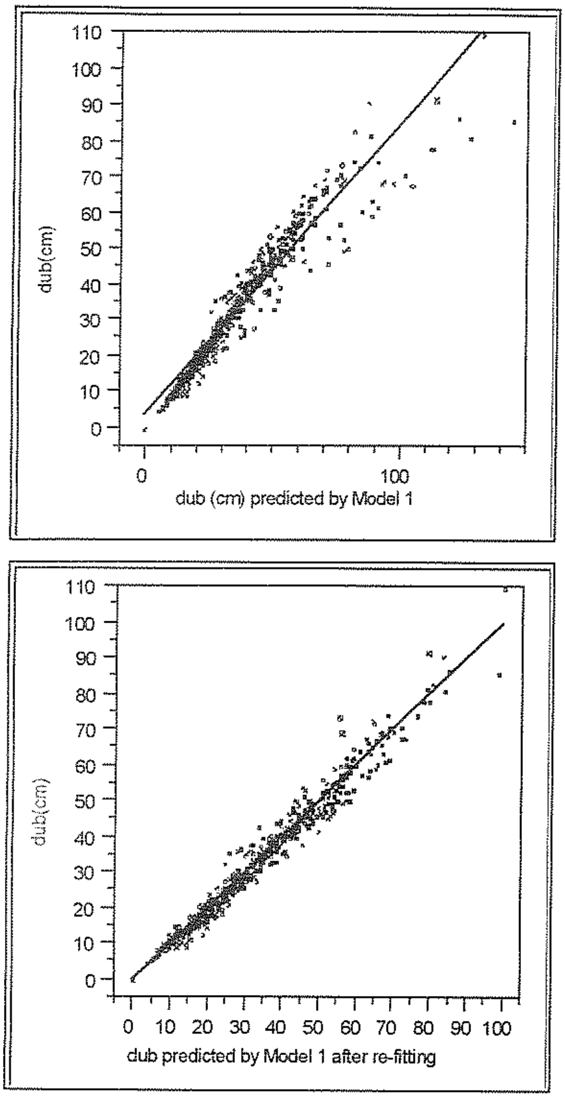


Figure 1: Scatterplots showing the distribution of underbark stem diameters predicted by model (1), relative to actual stem diameters, before (above) and after (below) the model was fitted to the data used in this study. Models (2)-(6) displayed similar distribution of actual vs predicted diameters.

The bias and imprecision associated with stem diameter predictions of the models tested could not automatically be attributed to inappropriate model form, as there were several factors which clearly challenged such a conclusion. Firstly, the form of the nonlinear (variable-form) models derived by Muhairwe (1995), was found by Kozak and Smith (1993) to be highly suitable for describing the shape of a tree stem. Secondly, the performance of the models tested was similar, despite differences in model form. The similarity in model performance was evident in three characteristics of each model's prediction errors: heterogenous error variance, non-normal error distribution and error correlation (see Figure 2). Each of these error characteristics violates one of the assumptions underlying the standard least squares procedure, which was used to derive model parameters. Moreover, when the standard least squares procedure was used to derive new model parameters, based on the data used in this study, none of these assumption violations were eliminated, or visibly reduced. Consequently, the poor performance of the models tested can, at least partially, be attributed to the inadequacy of their parameter estimates, rather than model form.

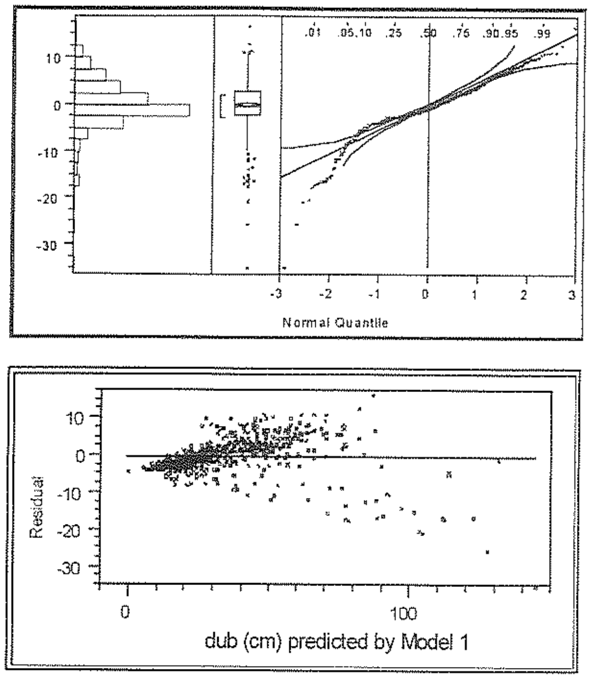
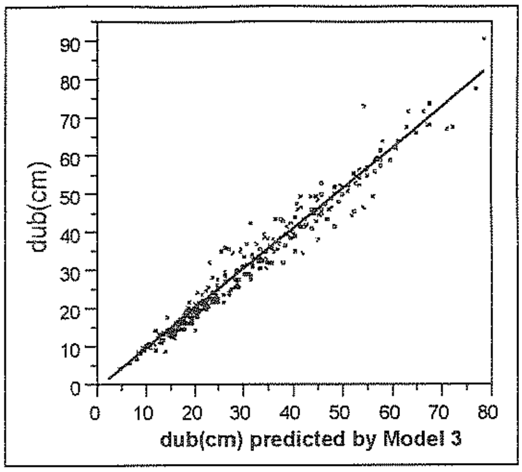


Figure 2: Scatterplots showing a normal quantile plot (above) indicative of non-normal error distribution, and heterogenous error variance (below). Both of these characteristics were displayed by each of the six models tested. Error correlation was determined through the Durbin-Watson coefficient.

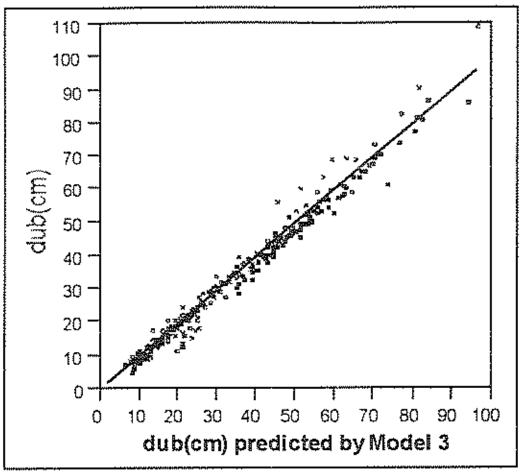
The proposal that the parameters of the taper models tested in this study do not adequately reflect data characteristics, is supported by the similarity between the set of data collected for this research and that on which the models were based. Both of these samples contain stem diameter measurements that correspond to a single tree, rather than several separate trees. This observation is important, since it asserts that one of the assumption violations mentioned earlier, namely correlation between model errors, was one of the factors responsible for the inadequacy of the models' *current* parameter estimates. Error correlation occurred because correlation between diameters corresponding to a *single tree* stem was not taken into account during the process used to estimate model parameters (standard least squares procedure, followed by nonlinear modelling). A procedure which takes data correlation into account and is therefore more appropriate, is two-stage modelling. During the first stage of this procedure, a taper model is fitted to each tree *individually* (accounting for correlated diameters) and the resulting parameters are related to tree and stand characteristics (Newberry and Burkhart, 1985). Consequently, when two-stage modelling was used to estimate the parameters of model (3), both bias and precision of model predictions were markedly improved (see Figure 3). It is also important to note that two-stage modelling resulted in a more normal distribution of error and homogenous, rather than heterogenous error variance. In other words, parameters estimated through a two-stage modelling procedure are more reliable than those obtained through nonlinear fitting, because they are associated with fewer (or less serious) violations of assumptions underlying the standard least squares procedure.

In addition to being a more appropriate method of describing the data used in this study, two-stage modelling was also the only method which could be used to examine the influence of soil type, altitude, point basal area and tree dominance on stem taper of *E.pilularis*. This is so because point basal area and tree dominance are tree, rather than stand characteristics, which therefore had to be examined at the tree level. On the other hand, soil type and altitude did not appear to influence the average stand taper of *E.pilularis*, as was evident from the uniform distribution of each model's prediction errors with respect to both of these stand attributes.

The process of relating the parameters of model (3) to variables describing tree and stand characteristics, revealed that both form and taper of individual *E.pilularis* trees are more strongly related to tree, rather than stand characteristics (see Appendix). This finding was confirmed by the fact that incorporating only point basal area, tree dominance, soil type and altitude into model parameters failed to improve its predictions. However, each of these four variables was significantly related to either form or taper parameters, when observed in the absence of any other variables (see Table 1). This indicates that the growing conditions surrounding *E.pilularis* trees influence their rate of taper, as well as form.



(a)



(b)

Figure 3: Scatterplots showing the distribution of model (3) diameter predictions versus actual diameters, subsequent to applying the model to that half of the sample which was not used to estimate model parameters. Model parameters in (a) were obtained through the standard least squares procedure and nonlinear fitting, while model parameters in (b) were derived through the two-stage modelling process.

In biological terms, the results of two-stage modelling carried out in this study show that the rate of taper of individual *E.pilularis* trees increases, the less dominant the tree, and as point basal area increases (Table 2). The finding that lower point basal area is associated with higher rates of taper, is supported by the fact that larger trees have high rates of taper, and are usually surrounded by less competition (Gray, 1956). The same reasoning fails to support the notion that less dominant trees have higher rates of taper, since larger trees, which have higher rates of taper, tend to be more dominant. However, it is worthwhile noting that tree dominance was subject to judgment variation (McRoberts *et al.*, 1994), as it was estimated visually. With respect to altitude and soil type, the results indicate that *E.pilularis* trees growing in earth soils and at high altitudes have higher rates of taper than those growing in podzolic soils and at low altitudes.

parameter	altitude	point basal area	tree dominance	soil type
a_0	ns	ns	<0.0001	0.0023
b_1	<0.0001	<0.0001	ns	ns
b_2	ns	<0.0001	ns	0.0400
b_3	0.0002	ns	<0.0001	ns

ns = not significant

Table 1: Significance levels (probability>F) of altitude, point basal area, tree dominance and soil type, in terms of estimating parameters shown. The values shown here are based on the sequential sums of squares, obtained by relating each parameter to all of the above stand variables (through the standard least squares procedure, *second stage* of two-stage modelling).

model 3 parameter	tree dominance	soil type	point basal area	altitude
a_0 (taper)	0.0201	-0.0134	-0.0007	0.0119
b_1 (form)	-0.0028	-0.0014	0.0024	-0.5354
b_2 (form)	0.0002	0.0017	0.0002	0.0006
b_3 (form)	0.0806	-0.0075	-0.0012	0.0479

Table 2: Parameter estimates obtained by relating model 3 taper and form parameters (each of the four parameters represents a separate *y* variable in the second stage of the two-stage modelling process) to tree dominance, point basal area, soil type and altitude, through the standard least squares procedure.

4. CONCLUSIONS AND RECOMMENDATIONS

The first objective of this research was to assess the adequacy of the existing *E.pilularis* taper models described in Muhairwe's (1995) study. In terms of this objective, the analyses showed that stem diameter predictions of the models were both biased and imprecise. Based on the data used in this study, this could mostly be attributed to data correlation, which was not taken into account during the process of estimating model parameters. This, in turn, resulted in correlated model errors that violated one of the assumptions underlying the *validity* of the procedure on which parameter estimates were based. Given that the data on which the models were based was similar, the results of this study strongly suggest that two-stage modelling is a more appropriate method to use for this purpose. Two-stage modelling eliminates the problem of error correlation, because it takes into account the fact that stem diameter measurements on which the models were based, do not all correspond to separate trees.

The second objective of this research was to try and improve model performance by incorporating into the analysis four factors: point basal area, tree dominance, soil type and altitude. This objective was achieved through the process of two-stage modelling, in other words, by isolating the influence of these variables on the rate of taper of individual *E.pilularis* trees. The results of this process indicated that each of these factors significantly influences either form or taper of *E.pilularis*. However, soil type and tree dominance are not easy to measure and may therefore reduce model performance from a practical viewpoint. Further research in this area is recommended, based on a wider range of data. This can both improve model predictions and allow a single model to be applied to a wider range of circumstances, making it more practical to use.

Finally, testing the influence of tree *species* on eucalypt taper represents an additional recommendation for future research. Incorporating the effects of tree species into a given taper model could allow a single model to be applied to different eucalypt species, as well as increase the accuracy of model predictions.

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7. APPENDIX

Results of stepwise regression analysis applied to model (3) parameters a_0, b_1, b_2 and b_3. The variables used in this analysis included all of the independent variables from each of the six models tested, along with soil type, altitude, point basal area and tree dominance.

In the table below, TH=total tree height (m); SDP=stem deterioration point (m); X3=dbh*(H-1.3)*H; H=height from stem base (ground level) (m); X1=$(H-1.3)^2$; BA=point basal area; DH2=$(dbh/TH)^2$; BH=breast height (1.3m); and D1=*ln*dbh (*ln*=natural logarithm).

parameter a_0	**Sequential SS**	**Cp**	**R^2**
TH	0.223945	13.443	0.0758
SDP	0.045031	5.444	0.091
X3	0.022148	2.5259	0.0985
X1	0.029164	-1.95	0.1084
BA	0.023211	-5.104	0.1162
dbh	0.008462	-4.983	0.1191
dbh/TH	0.008448	-4.859	0.122
parameter b_1			
SDP	22.36262	159.57	0.1307
DH2	9.213017	114.94	0.1846
BA	8.789386	72.459	0.236
X3	6.174567	43.209	0.2721
(1-BH/TH)	3.32329	28.389	0.2915
dbh/TH	2.048595	20.021	0.3035
D1	2.215926	10.806	0.3164
X1	1.533367	5.0456	0.3254
TH	1.143808	1.2567	0.3321
$(H/TH)^2$	0.562724	0.4087	0.3354
parameter b_2			
BA	0.00326	8.4192	0.0546
SDP	0.000448	5.647	0.0621
X3	0.000568	1.5951	0.0716
(1-BH/TH)	0.00018	1.6727	0.0746
TH	0.000542	-2.099	0.0837
parameter b_3			
D1	2.876196	19.383	0.2607
dbh/TH	0.276771	0.6012	0.2858
X4	0.176474	-10.65	0.3018
X1	0.031465	-11.01	0.3047